EL1

Mathematics
Support Centre

Title: Introduction to circuits and Ohm's Law

Target: On completion of this worksheet you should be familiar with fundamental circuit parameters and Ohm's Law.

Introduction

The field of **Electrical Engineering focuses on** two main areas: **the distribution and use of electrical energy** (e.g. robots) **and information** (e.g. communication systems). Irrespective of the circuit's complexity or functionality, it is tested and analysed using fundamental circuit laws.

Electrons Flow and Current

The motion of "free" electrons in neutral state is random with no particular direction or speed. Electrons can move in a coordinated direction (flowing from the negative to the positive terminal) in a conductor. This is called dynamic electricity or electric current.
The electric current is defined as the rate of charge flow in an electric circuit, measured in Ampere (A).

$$I = \frac{charge}{time} = \frac{\Delta Q}{\Delta t}$$

$Q = ne$ for $e = 1.6 \times 10^{-19}$ C and n the number of electrons.

Resistance

Resistance is an electrical quantity measuring how much the material reduces the flow of electric current through it. It is measured in Ohm (Ω). A resistor is a passive component which provides resistance in the circuit. The resistance of a conductor R is directly proportional to the conductor length L (m) and inversely proportional to the cross-sectional area a (m^2).

$$R = \frac{\rho L}{a}$$

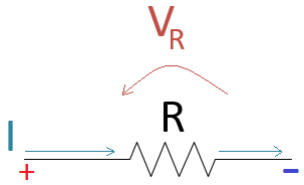
with ρ the resistivity of the material (Ωm).
Example: A piece of wire of 10m length with a diameter of 2.5 mm, has a resistivity of 14.8 n Ωm. What is the wire's resistance?

$$R = \frac{\rho L}{a} = \frac{\rho L}{\pi(\frac{d}{2})^2}$$

$$R = \frac{14.8 \times 10^{-9} \times 10}{\pi(\frac{2.5 \times 10^{-3}}{2})^2} = 3.769 \times 10^{-5}\ \Omega$$

Voltage

The energy transferred due to the passage of unit charge between two points in a circuit is called the potential difference or voltage, measured in Volt (V). The direction of current and the potential difference across a load in a circuit is opposite.

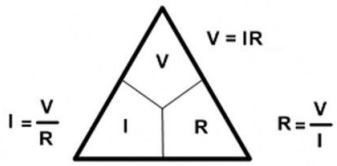


Ohm's Law

Ohm's Law states that the potential difference across a conductor is directly proportional to the current flowing through the conductor.

$$V = IR$$

where R is the resistance of the conductor.



Power

Power is defined as the amount of energy expanded in a fixed time measured in Watt (W).

$$P = \frac{\text{energy}}{\text{time}} = \frac{VQ}{t} = V\frac{Q}{t} = VI$$

In a circuit the power generated (by sources) is equal to the power dissipated (by circuit components).
The power dissipated in a resistor:

$$P_{diss} = VI$$

Using Ohms Law and substituting $V = IR$:

$$P_{diss} = I^2R \text{ or } P_{diss} = \frac{V^2}{R}$$

Energy

The energy required to drive a particular charge through the circuit is proportional to the quantity of charge and voltage measured in Joules (J).

$$E = QV \text{ or } E = Pt$$

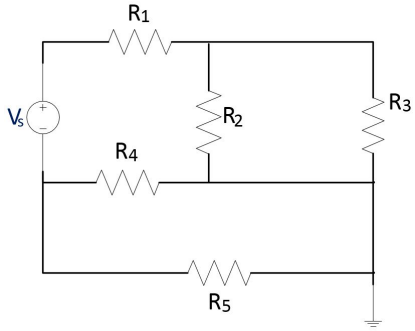
P.T.O.

Circuit topology

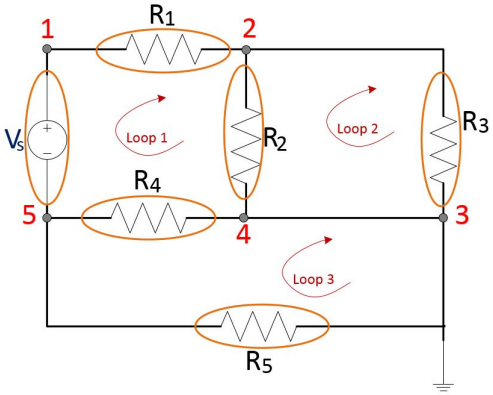
In order to solve and analyse electrical circuits it is essential to know a circuit's topology elements:

1. **Branch:** A branch is an element of a circuit having only two terminals.
2. **Node:** A node is the junction point of two or more branches and it refers to a point on the circuit where the voltage is the same.
3. **Closed Loop/Mesh:** A mesh is a closed loop (starting from one node and returning to it without passing by an intermediate node more than once) which does not contain any other loop within it.

Example: Determine the branches, nodes and loops in the circuit.



Solution: Firstly marking the branches followed by the nodes and lastly the loops.

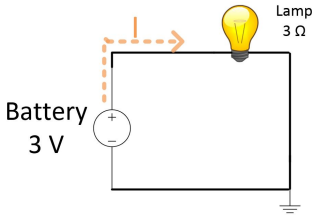


Exercises

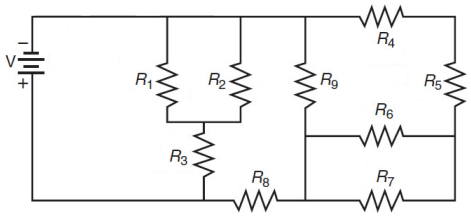
1. What is the voltage drop across an electric heater of 100 Ω resistance when the current is 0.1 A ?
2. Calculate the power dissipated by an electric lamp which draws 2.5 A from a 140 V line?
3. What is the power used by a 15 Ω electric heater when a voltage of 210 V is applied? How much energy it will consume in 2 hours?

Exercises(Cont.)

4. Find the current value in the following circuit.



5. Determine the number of branches, nodes and loops in the circuit below.



Solutions

1. Using Ohm's Law: $V_{drop} = IR$
   $V_{drop} = 0.1 \times 100 = 10$ V
2. Using the Power formula: $P = VI$
   $P = 140 \times 2.5 = 350$ W
3. Calculating the Power : $P = \dfrac{V^2}{R}$
   $P = \dfrac{210^2}{15} = 2940$ W
   Converting the time in seconds:
   $t = 7200$ s
   Calculating the Energy: $E = Pt$
   $E = 2940 \times 7200 = 21.168$ MJ
4. Using Ohm's Law: $I = \dfrac{V}{R}$
   $I = \dfrac{3}{3} = 1$ A
5. Branches $= 10$
   Nodes $= 9$
   Loops $= 5$