EL2

Mathematics
Support Centre

Title: Kirchhoff's Current and Voltage Laws

Target: On completion of this worksheet you should be able to apply Kirchhoff's laws.

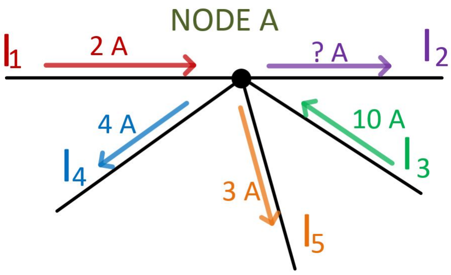
Introduction

Circuit analysis aims to calculate the voltage and current through a circuit component as well as the power consumed and generated. There are multiple methods of calculating these quantities but all have at their core the fundamental Kirchhoff's laws.

Kirchhoff's Current Law (KCL)

KCL states that the sum of currents entering a node (total I_{in}) is equal to the sum of currents leaving that node (total I_{out}), $I_{in} = I_{out}$.

Example: Calculate the value of I_2.



Solution: Writting KCL with all the currents entering node A on the LHS and exiting on the RHS.

$$I_1 + I_3 = I_2 + I_4 + I_5$$

Substituting the values:

$$2 + 10 = I_2 + 4 + 3$$
$$12 = I_2 + 7$$
$$I_2 = 5 \text{ A}$$

Kirchhoff's Voltage Law (KVL)

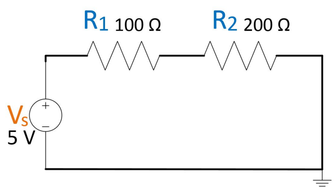
KVL states that the sum of the voltages (both sources and drops) around a single closed path or loop is equal to zero.

Note: The number of independent KVL equations in a circuit:

$$e = branches - nodes + 1 - I$$

where I is the number of current sources.

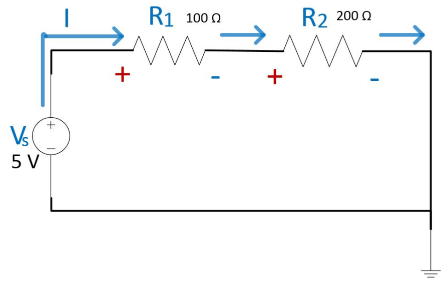
Example: Using KVL to find the value of the current.



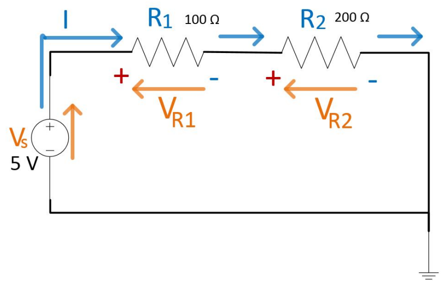
Solution

1. Calculate the number of independent equations and mark the direction of the current in the circuit.

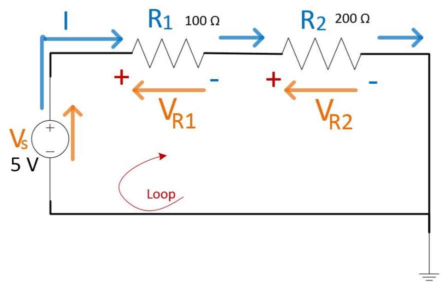
$$e = b - n + 1 - I = 3 - 3 + 1 - 0 = 1$$



2. Indicate the direction of the source voltage V_s and voltage drops on the resistors R_1 and R_2.
Note: Remember that the direction of the voltage drop is opposite to the current flow!



3. Choose a direction clockwise or anti-clockwise as $+ve$ through the loop. In this example, we will choose clockwise as $+ve$.



4. Write the KVL equation by starting with the voltage source and gradually following each component. If the arrow direction of a voltage is opposite to your chosen $+ve$ direction then that voltage is negative. Otherwise, consider it positive.

$$V_s + (-V_{R1}) + (-V_{R2}) = 0$$
$$V_s - V_{R1} - V_{R2} = 0$$

P.T.O.

Solution(Cont.)

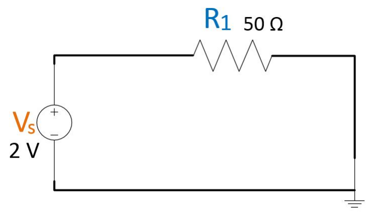
5. Expand each voltage drop V_{R1} and V_{R2} using Ohm's Law and substitute all known variables.

   Note: The same current I flows through both resistors R_1 and R_2.

$$V_s - I \times R_1 - I \times R_2 = 0$$
$$V_s - I \times (R_1 + R_2) = 0$$
$$V_s = I(R_1 + R_2)$$
$$I = \frac{V_s}{R_1 + R_2} = \frac{5}{100 + 200} = 0.017 \text{ A.}$$

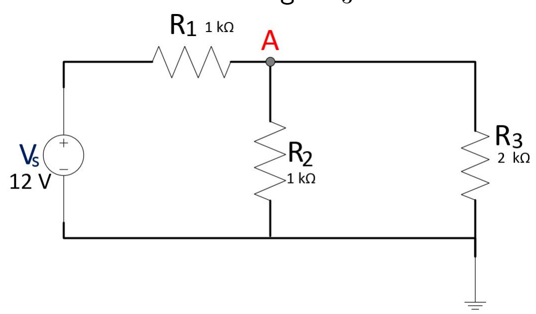
Exercises

1. For the following circuit calculate:

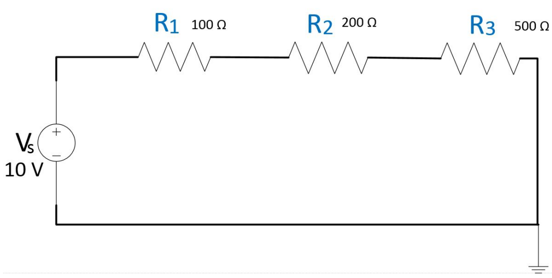


   (a) potential difference across R_1;
   (b) power dissipated by the resistor;
   (c) energy absorbed in 1 minute.

2. For the following circuit, calculate the source current and the one through R_3.

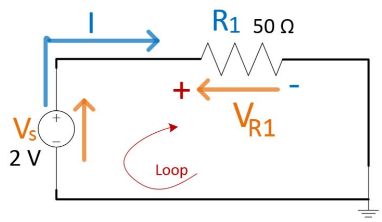


3. For the circuit below calculate the voltage drop across R_2.



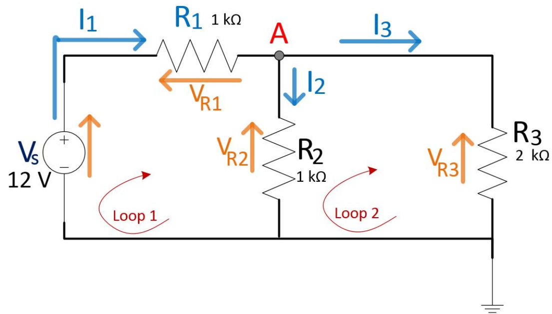
Solutions

1. Considering this is a simple circuit, it can be solved by either using Ohm's Law or KVL. Marking the circuit and writing the KVL equation: $V_s - V_{R1} = 0$.



   (a) $V_s = V_{R1} = 2$ V
   (b) $P = VI = \frac{V^2}{R} = \frac{4}{50} = 0.08$ W
   (c) $E = Pt = 0.08 \times 60 = 4.8$ J

2. Determining the number of independent KVL equations:
$$e = b - n + 1 - I = 4 - 3 + 1 - 0 = 2$$



$$Loop\ 1:\ V_s - V_{R1} - V_{R2} = 0$$
$$Loop\ 2:\ V_{R2} - V_{R3} = 0$$

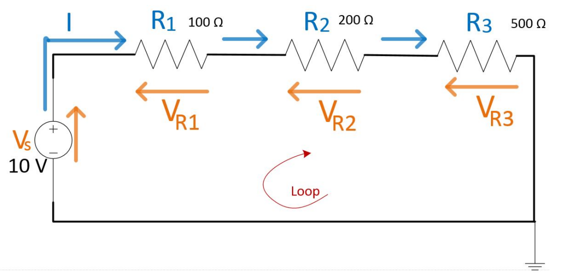
   Writing the KCL for Node A: $I_1 = I_2 + I_3$ and substituting it to $Loop\ 1$:

$$V_s - (I_2 + I_3)R_1 - I_2R_2 = 0$$
$$V_s - I_2(R_1 + R_2) - I_3R_1 = 0$$

   Solving the equation leads to: $I_2 = 2I_3$.

   Final answers: $I_3 = 0.0024$ A and $I_1 = 0.0072$ A.

3. Writing the KVL eq.: $Vs - V_{R1} - V_{R2} - V_{R3} = 0$.



$$V_s = I(R_1 + R_2 + R_3)$$
$$I = \frac{V_s}{R_1 + R_2 + R_3}$$
$$I = \frac{10}{100 + 200 + 500} = 0.0125 \text{ A}$$

   Calculating the voltage across R_2:

$$V_{R2} = IR_2 = 2.5 \text{ V}$$