EL3

Mathematics Support Centre

Title: Voltage and Current Divider Rules

Target: On completion of this worksheet you should be able to find the equivalent resistance of a circuit and apply the current and voltage divider rules.

Introduction

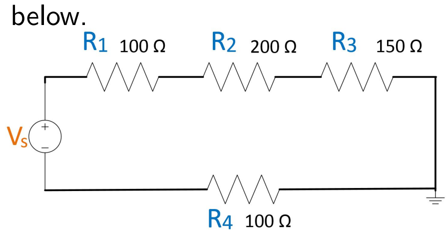
The current and voltage divider rules are useful for calculating circuit parameters without the need of writing the KVL equations and doing mesh analysis. Generally, finding the equivalent resistance of a circuit is essential for this approach and it allows us to calculate the source current which then helps finding the wanted quantity.

Note: The equivalent resistance of a circuit is the total resistance R_{eq}.

Elements in Series

If two or more elements are connected with the same current I carrying through them then the elements are **in series**.

Example: Calculate the equivalent resistance for the circuit below.



Solution:

1. Analysing the currents in the circuit. There is a single loop with one current flowing through all 4 resistors so the elements are in series.

2. Using the generalised formula for n-components:
$$R_{eq} = R_1 + R_2 + R_3 + ... + R_n$$
Applying it to this circuit:
$$R_{eq} = R_1 + R_2 + R_3 + R_4$$
$$R_{eq} = 100 + 200 + 150 + 100 = 550\ \Omega$$

Voltage Divider Rule

The voltage divider rule states that when resistors are connected in series, the voltage is divided. The voltage drop on a resistor R_i can be calculated using the formula:

$$V_{R_i} = V_s \times \frac{R_i}{R_1 + R_2 + ... + R_n}$$

Example: Using the circuit above with $V_s = 10$ V, calculate the voltage V_{R_2} using two different methods (Ohm's Law and the Voltage Divider Rule).

Solution

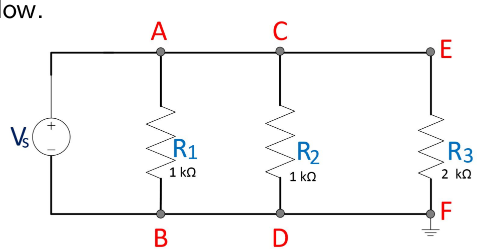
1. Calculating the equivalent resistance.
$$R_{eq} = 550\ \Omega$$
Applying Ohm's Law to calculate the supply current:
$$I = \frac{V_s}{R_{eq}} = \frac{10}{550} = 0.018 \text{ A}$$
Calculating the voltage drop on R_2:
$$V_{R_2} = IR_2 = 0.0018 \times 200 = 3.636 \text{ V}$$

2. Using the Voltage Divider Rule:
$$V_{R_2} = V_s \times \frac{R_2}{R_1 + R_2 + R_3 + R_4} = V_s \times \frac{R_2}{R_{eq}}$$
$$V_{R_2} = 10 \times \frac{200}{550} = 3.636 \text{ V}$$

Elements in Parallel

If there is more than one current path between two or more elements and if the absolute value of the potential difference V between them is equal, then the elements are **in parallel**.

Example: Calculate the equivalent resistance for the circuit below.



Solution

1. Analysing the circuit, we observe that the voltages V_{AB}, V_{CD} and V_{EF} are equal. Therefore, the three resistors are connected in parallel.

2. Using the generalised formula for n-components:
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$
Applying it:
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{1} + \frac{1}{2}$$
$$R_{eq} = 0.4 \text{ k}\Omega$$

Current Divider Rule

The current divider rule states that when resistors are connected in parallel, the current is splitting between the branches. The current through a resistor R_i is given by:

$$I_{R_i} = I \times \frac{R_{eq'}}{R_i + R_{eq'}}$$

where $R_{eq'}$ is the equivalent resistance of the resistors without R_n.

Example: Using the previous parallel circuit with $V_s = 10$ V, calculate the current through R_3 using two different methods (Ohm's Law/KVL and the Current Divider Rule).

Solution

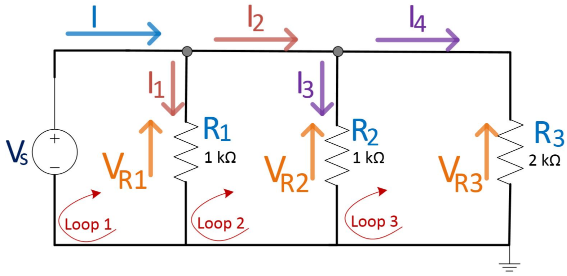
1. Calculate the total equivalent resistance

$$R_{eq} = 400\ \Omega$$

Use Ohm's Law to find the source current I:

$$I = \frac{V_s}{R_{eq}} = \frac{10}{400} = 0.025 \text{ A}$$

Applying KVL and KCL for all three loops:



$$I = I_1 + I_2 \text{ and } I_2 = I_4 + I_3$$
$$Loop\ 1 : V_s \quad - V_{R1} = 0$$
$$Loop\ 2 : V_{R1} - V_{R2} = 0$$
$$Loop\ 3 : V_{R2} - V_{R3} = 0$$

Solving the equations gives $I_4 = 0.005A$

2. Calculate the equivalent resistance of R_1 and R_2:

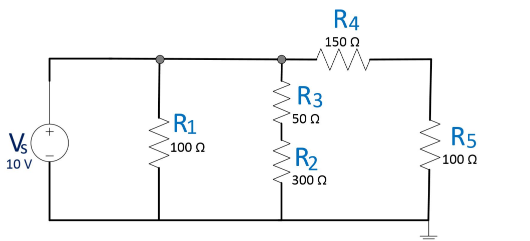
$$\frac{1}{R_{eq'}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{1} + \frac{1}{1}$$
$$R_{eq'} = 0.5 \text{ k}\Omega$$

Applying the formula:

$$I_4 = I \times \frac{R_{eq'}}{R_{eq'} + R_3} = 0.025 \times \frac{0.5}{0.5 + 2}$$
$$I_4 = 0.005 \text{ A}$$

Note: The two methods are equivalent with the second solution being much quicker compared to using Kirchhoff's Laws.

Exercise: Calculate the voltage drop across R_3 using: Kirchhoff's Laws and Current/Voltage Divider rules.



Solution

1. Calculate R_{eq} and supply current I.

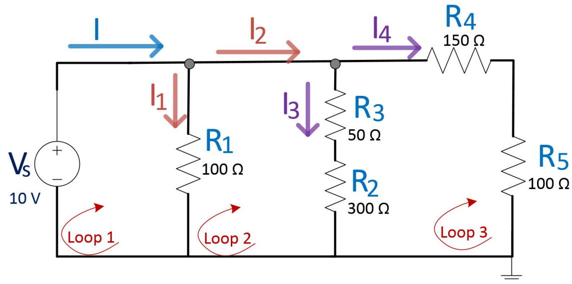
$$R_{45} = R_4 + R_5 = 250\ \Omega$$
$$R_{23} = R_2 + R_3 = 350\ \Omega$$

The circuit consists of three main resistors R_1,R_{23} and R_{45} connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{23}} + \frac{1}{R_{45}}$$
$$R_{eq} = 59.3\ \Omega$$

Calculate the supply current:

$$I = \frac{V_s}{R_{eq}} = \frac{10}{59.3} = 0.168 \text{ A}$$



Write the KVL and KCL equations for $Loop\ 1$ and $Loop\ 2$:

$$I \ = I_1 + I_2$$
$$I_2 = I_3 + I_4$$
$$Loop\ 1 : V_s \quad - V_{R1} = 0$$
$$Loop\ 2 : V_{R1} - V_{R3} - V_{R2} = 0$$
$$Loop\ 1 : V_s \quad - I_1 R_1 = 0$$
$$Loop\ 1 : I_1 = \frac{V_s}{R_1} = 0.1 \text{ A}$$
$$Loop\ 2 : I_1 R_1 - I_3 R_3 - I_3 R_2 = 0$$
$$Loop\ 2 : I_3 = \frac{I_1 R_1}{R_3 + R_2} = 0.028\ A$$

Calculate the voltage drop:

$$V_{R3} = I_3 R_3 = 1.428 \text{ V}$$

2. Knowing that the voltage distributed across the branches is equal to the voltage supply, we can assume that $V_s = V_{R1} = V_{R23} = V_{R45}$.

Applying the Voltage Divider rule across the branch for V_{23}:

$$V_{R3} = V_s \times \frac{R_3}{R_3 + R_2}$$
$$V_{R3} = 10 \times \frac{50}{350} = 1.428 \text{ V}$$

Note: There are multiple options for solving this question using the Current and Voltage divider rules. This means that there is not a fixed way for solving it. You can solve it using a different method as long as the laws are correctly applied.