EL5

Mathematics Support Centre

Title: Capacitors and Inductors

Target: On completion of this worksheet you should be familiar with the basic concepts around capacitors and inductors.

Introduction

The three main passive components used in circuits are resistors, capacitors and inductors.Resistors, whether used in DC or AC circuits, will always have the same resistance value irrespective of the supply frequency. On the other hand, capacitors and inductors have a different type of AC resistance known as reactance. The reactance value depends on the frequency of the supply source.

Note: A theoretically ideal capacitor or inductor does not have any resistance. However, in the real world they always have some resistive value no matter how small.

Capacitors

A capacitor is a passive component which has the ability to store energy in the form of electrical charge Q, similar to a small battery. Capacitance is measured in Farad (F).

$$Q = CV_c$$

with V_c the voltage drop across the capacitor.

In a DC circuit, capacitors always have infinite impedance Z_c acting like an open-circuit. In an AC circuit, only at very high frequencies, capacitors have zero impedance, similar to a short-circuit.

The reactance and impedance are defined as:

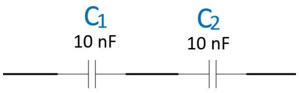
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ and } Z_c = -jX_c.$$

Equivalent capacitance

Similar to resistors, capacitors can be connected in series and parallel.

1. For n-components connected in **series** the equivalent capacitance is: $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_n}$.

   Example: Calculate the equivalent capacitance of the two capacitors C_1 and C_2.

   Solution

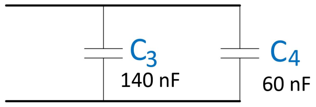
   $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{10}$$

   $$C_{eq} = 5 \text{ nF}$$

2. For n-components connected in **parallel** the equivalent capacitance is:

   $$C_{eq} = C_1 + C_2 + \ldots + C_n$$

   Example: Calculate C_{eq} of the two capacitors C_3 and C_4.

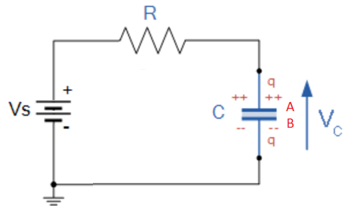
   

   Solution

   $$C_{eq} = 140 + 60 = 200 \text{ nF}$$

Charging and discharging a capacitor

In an AC circuit, a capacitor charges during the positive half-cycle and discharges during the negative-half cycle. For example, it can be charged by a simple RC circuit:



If the voltage across the capacitor is initially zero then the current $i(t)$ will flow around the circuit such that the charge $q(t)$ builds up on the capacitor's plates. When the voltage A to B will be approximately equal the voltage of the source then current will stop flowing.

Inductors

An inductor is a passive component, unlike a capacitor that oppose a change of voltage across their plates, opposes the changes of current by inducing a magnetic field. For a steady DC current, the inductor behaves as a short circuit. Inductance is measured in Henry (H).

$$V_L = L\frac{di}{dt}$$

The voltage V_L is proportional to the rate of change of current. If i is increasing, then V_L rises to oppose the increase in current.

The reactance and impedance are defined as:

$$X_L = \omega L = 2\pi f L$$

$$Z_L = jX_L = j\omega L$$

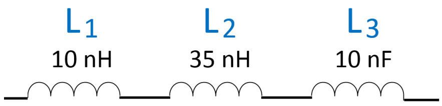
Equivalent inductance

Similar to capacitor and resistors, inductors can be connected in series and parallel.

1. For n-components connected in **series** the equivalent inductance is:

$$L_{eq} = L_1 + L_2 + ... + L_n$$

Example: Calculate the equivalent inductance.



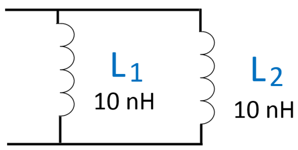
Solution:

$$L_{eq} = L_1 + L_2 + L_3$$
$$L_{eq} = 10 + 35 + 10 = 55 \text{ nF}.$$

2. For n-components connected in **parallel** the equivalent inductance is:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$$

Example: Calculate the equivalent inductance.



Solution:

$$\frac{1}{L_{eq}} = \frac{1}{10} + \frac{1}{10}$$
$$L_{eq} = 5 \text{ nF}$$

Exercises

1. A capacitor of $100\ \mu$F is charged up to 5 V. What is the magnitude of the charge on each plate of the capacitor?

2. A current of 5 mA is passed through a $1\ \mu$F capacitor for 1 ms. If the capacitor was initially uncharged, what is the final voltage across the capacitor?

3. A circuit comprises of three parallel-connected, 10μH inductors, with a 5μH inductor in series. Find the single inductor value which will replace this combination.

4. Within a circuit, the inductor L is connected to an AC power supply of 10 V with the frequency 200 kH. Considering the inductance is 5 nH, what is the reactance value for L?

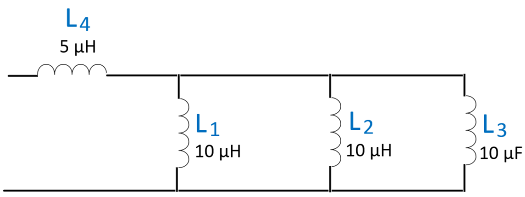
5. A circuit comprises of two parallel-connected $10\ \mu$F capacitors, in series with a $10\ \mu$F capacitor. Each capacitor can withstand 6 V before breaking down. How much voltage can the combination withstand?

Solutions

1. $Q = CV = 100 \times 10^{-6} \times 5 = 5 \times 10^{-4} = 0.5$ mC. The charge on each plate is equal to 0.5 mC.

2. $\Delta V = i\frac{\Delta t}{C} = 5 \times 10^{-3} \times \frac{10^{-3}}{10^{-6}} = 5$ V

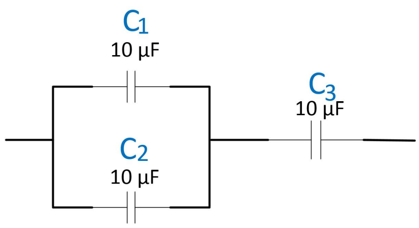
3. Draw the circuit diagram.



$$\frac{1}{L_{123}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$
$$L_{123} = 3.33\ \mu\text{H}$$
$$L_{eq} = L_4 + L_{123} = 8.33\ \mu\text{H}$$

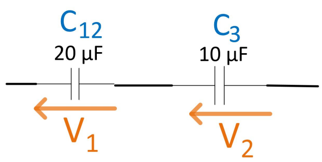
4. $X_L = 2\pi f L = 2\pi \times 200 \times 10^3 \times 5 \times 10^{-9} = 6.28$ mΩ

5. Draw the circuit diagram.



$$C_{12} = C_1 + C_2 = 20\ \mu\text{F}$$

Note: Remember that the charge is the same for capacitors in series.



$$Q = V_1 C_{12}$$
$$Q = V_2 C_3$$
$$\frac{V_1}{V_2} = \frac{C_3}{C_{12}} = \frac{10}{20} = \frac{1}{2}$$
$$V_2 = 2V_1$$

The maximum voltage that an individual capacitor can withstand is 6 V. Considering V_2 was found to be larger than V_1, we can set V_2 to be the maximum allowed voltage. Therefore, $V_1 = 3$ V.
The entire configuration can withstand:

$$V_{max} = V_1 + V_2 = 6 + 3 = 9 \text{ V}$$