E5

Mathematics Support Centre

Title: Friction

Target: On completion of this worksheet you should be able to analyse motion of bodies in contact.

Friction

Friction is a force resisting the relative movement of two surfaces in contact. The magnitude of friction is given by $F_f \leq \mu F_n$, where μ is the **coefficient of friction** and F_n is the normal force to the surface. On a surface, friction always points in the direction opposing the direction of motion. If the body is at rest, friction will point in the opposite direction of the forces acting on the body. Friction is a **dissipative force**, meaning that it causes energy loses in the forms of heat, structural changes (bending of objects), or acoustic waves. This is the reason why we rub our hands in winter when we're cold!

Static and dynamic friction coefficient

Depending on the situation of bodies in contact, we distinguish two types of coefficients of frictions.
The static friction coefficient, μ_s, is used when two surfaces are at rest. Then $F_f \leq \mu_s F_n$.
The dynamic friction coefficient, μ_k, is used when two surfaces are in relative motion. Then $F_f = \mu_k F_n$
The idea of 2 different values for the coefficient of friction might be at first surprising, but we can actually observe this phenomenon in every-day life. For example, as we try to push a bed, we notice that it resists motion. This is when the static friction coefficient is affecting the system. As we reach a critical force, a sudden transition between the static and dynamic friction occurs and the bed begins to move. A remarkable result is that the force required to keep the bed in motion is **less** than the force required to initiate the motion, i.e. $\mu_k < \mu_s$.

The laws of friction

The three empirical observations that led to the formulation of friction are

1. The force of friction is directly proportional to the applied load.
2. The force of friction is **independent of the area** of contact of bodies.
3. Kinetic friction is **independent of the sliding velocity**.

Work of friction

The work of friction in a rectilinear motion is defined to be $W_f = F_f \Delta x = \mu F_n \Delta x$, where Δx is the distance covered by the body. As an immediate result we have that *the static friction does no work*, since $\Delta x = 0$. Most of the times we associate W_f with the **heat** produced due to friction. Work and heat are measured in Joules.

Exercises

1. A body of mass $m = 2$kg is being pushed horizontally by a force of magnitude $F = 30$N on a flat surface. ($g = 9.81$m/s^2).
   a) Given that $\mu_s = 1.1$ find out if the body will move.
   b) Given that $\mu_k = 0.9$, find the acceleration of motion.
   c) The motion lasts for 5s. What is the heat emitted as a result of the friction?
   d) Write down the equation that has to be solved in order to find the minimum angle α (between the force and the surface) that the force could be applied at without causing the motion of the object. Use numerical methods or otherwise to solve it.

2. Two cubical bodies are each being pushed with a slowly increasing force, tangential to the surface that they are placed on. The coefficient of friction is the same for each body. Initially the bodies are at rest. The density is $\rho_1 = 0.8$kg/m^3 for body 1 and $\rho_2 = 2.7$kg/m^3 for body 2, the side lengths are 0.3m for body 1 and 0.2m for body 2
   a) Find out which body will be the first to initiate motion.
   b) What can you say about the surface area of the bodies and its effect on the process?
   c) What is the total work done by friction on each body before the motion begins?

3. For a car driving on a flat surface with a coefficient of static friction of $\mu_s = 0.75$ and a coefficient of kinetic friction of $\mu_k = 0.67$; determine the maximum starting acceleration with and without "burning rubber". How do these two methods of starting a car compare? ($g = 9.81$m/s^2)

P.T.O.

Answers

1. a) We have $F_f \approx 21.6$ N$< F$, so the body will move.
   b) $a \approx 6.17$m/s^2.
   c) $W_f = \mu_k mg\frac{at^2}{2} \approx 1361.88$J.
   d) If the force was applied at an angle, the driving force becomes $F_d = F\cos(\alpha)$. The maximum value the static friction can take becomes $F_f = \mu_s(mg + F\sin(\alpha))$. When $F_f = F_d$ the motion is just about to begin. We therefore have to solve $\mu_s(mg+F\sin(\alpha)) = F\cos(\alpha)$. Using numerical software (e.g. MATLAB's 'solve' command from the symbolic package) we get $\alpha \approx 13.33°$.

2. a) Both bodies will begin to move at the same time, since they have equal mass.
   b) The surface area is different between the two bodies and it has no effect on the motion.
   c) It's 0, since $\Delta x = 0$.

3. Without 'burning rubber' the tyres don't slip, so we have $a_1 = \frac{mg\mu_s}{m} \approx 7.35$m/s^2. With 'burning rubber' we get $a_2 = \frac{mg\mu_k}{m} \approx 6.57$m/s^2. Evidently, it's better not to 'floor' a car since some energy is lost into heat.