E6

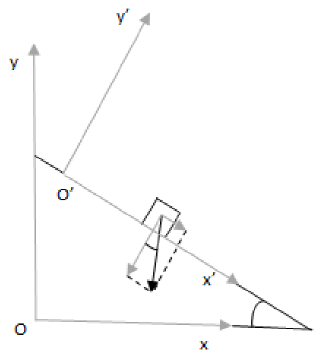
Mathematics Support Centre

Title: Motion along an inclined plane

Target: On completion of this worksheet you should be able to solve problems containing a ramp.

Coordinate system transformation

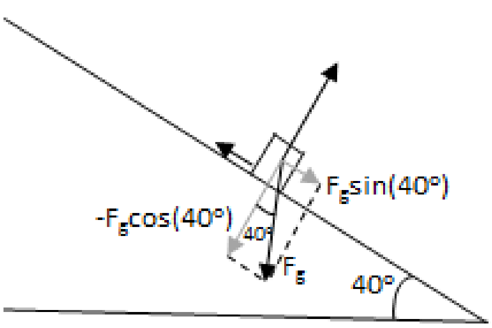
The choice of coordinates with which we describe an event can lead to many simplifications in the properties of a system. For example, let us consider the motion of a body on a frictionless, inclined plane, as shown in the figure below.



In the coordinates system (x, y) each component of the of position vector $\mathbf{r}(t) = (x(t), y(t))$ changes as time progresses. As viewed from these coordinates the motion seems 2D. However, let us consider the description of the motion in the coordinates (x', y'). Here we see, that $y'(t) = 0$, and therefore $\mathbf{r}'(t) = (x'(t), 0)$. The motion is 1D when viewed from that perspective! How do we change from one description to another?

Forces on the inclined plane

In the diagram below we can see all the forces acting on a body that moves with a constant velocity on a ramp or is stationary. We have that the normal force to the surface has value $F_n = F_g \cos(\alpha)$, the friction becomes $F_f \leq F_g \cos(\alpha)\mu_s$ and the driving force is $F_d = F_g \sin(\alpha)$.



Usage

Because of the way the forces are projected onto the ramps plane, we can use it to lower the amount of force required to pull something up! By doing so, however, we increase the distance required for transportation so that, for frictionless surfaces, the work done in two cases remains the same. The work needed to be done increases if there is friction.

Exercises

1. An object weights 20 N when weighted on a a flat surface. What would be the normal force if this object was placed on a ramp of slope $45°$?

2. In order to measure the coefficient of static friction μ_s between two materials, an experiment can be performed, where a body is placed on a ramp and then the slope of the ramp is is being slowly increased until a critical slope α_c is reached, where the body starts to slide. The formula relating the two is of the form $\mu_s = f(\alpha_c)$. Find $f(\alpha_c)$.

3. A box can be pulled down the ramp with a constant velocity when pushed with a force $\mathbf{F_1}$ parallel to it. Also, a box can be pulled up the ramp at a constant speed with a force, $|\mathbf{F_2}| = 10|\mathbf{F_1}|$, pointing in the opposite direction. The slope of the ramp is $\alpha = 10°$. Determine the coefficient of kinetic friction.

4. A box is placed on a wooden ramp. The mass of the box is 2 kg. The angle of the ramp with the horizontal is $\alpha = 30°$.
   a) Calculate the acceleration down the ramp assuming it's frictionless.
   Given $\mu_s = 0.5$ and $\mu_k = 0.3$:
   b) Calculate the acceleration down the ramp including friction.
   c) What supporting force parallel to the ramp would be needed to prevent the box from sliding?
   d) The box slides down the wooden ramp freely for 2 meters then it continues on a nearly frictionless surface of ice for 5 meters, to finally start climbing a 1 m long wooden ramp with the same μ_k, of the slope of $45°$. Will the box jump the ramp?

P.T.O.

Answers

1. $F_n = 10\sqrt{2}$ N≈ 14.14 N perpendicular to the surface, in the upward direction.

2. When the box is just about to start sliding we have $mg\sin(\alpha_c) = \mu_s mg\cos(\alpha_c)$. Therefore $\mu_s = \frac{\sin(\alpha_c)}{\cos(\alpha_c)} = \tan(\alpha_c)$, i.e $f(\alpha_c) = \tan(\alpha_c)$

3.

$$\begin{cases} F_1 + mg\sin(\alpha) - \mu_k mg\cos(\alpha) = 0 \\ -F_2 + mg\sin(\alpha) + \mu_k mg\cos(\alpha) = 0 \end{cases}$$

$$\begin{cases} F_1 + mg\sin(\alpha) - \mu_k mg\cos(\alpha) = 0 \\ -10F_1 + mg\sin(\alpha) + \mu_k mg\cos(\alpha) = 0 \end{cases}$$

So, $\mu_K = \frac{11}{9}\frac{\sin(\alpha)}{\cos(\alpha)} \approx 0.22$.

4. a) $F = mg\sin(\alpha)$. $a = \frac{1}{2}g \approx 4.9$ m/s^2.
b) Firstly we check if the body will even start moving $F_d = mg\sin(\alpha) \approx 9.81$ N$>$ $\mu_s mg\cos(\alpha) \approx 8.5$ N. Now we can find the acceleration, which is $a = g\sin(\alpha) - \mu_k g\cos(\alpha) \approx 2.36$ m/s^2.
c) The force needed to stop the box would be $-ma \approx -4.71$ N while it's in motion, however the force required to **support** the box is only $F \approx 1.31$ N.
d) On the first ramp we have that $s_1(t) = \frac{1}{2}a_1 t^2$, where $a_1 = g\sin(30°) - \mu_k g\cos(30°) \approx 2.36$ m/s^2. We need to find the final velocity of the box at the bottom of the ramp. To do that we find the time t_1 it takes for the box to slide $l_1 = 2$ meters: $l_1 = \frac{1}{2}a_1 t_1^2$. So, $t_1 = \sqrt{\frac{2l_1}{a_1}} \approx 1.3$ s. Then, at any time t on the first ramp we have $v(t) = a_1 t$, so the final velocity with which the box will leave the ramp is $v(t_1) = \sqrt{2l_1 a_1} \approx 3.07$ m/s. With this velocity, the box will enter the ice. Since there are no forces acting on the box on the ice, the box will move in a rectilinear motion and it will enter the second ramp with the same velocity. On the second ramp we have the equation of motion $s_2(t) = \frac{1}{2}a_2 t^2 + \sqrt{2l_1 a_1}t$, where $a_2 = -g\sin(45°) - \mu_k g\cos(45°) \approx -9.02$ m/s^2. All we need to check is if there exist values of t that satisfy $s_2(t) = 1$. i.e. we seek solutions to $\frac{1}{2}a_2 t^2 + \sqrt{2l_1 a_1} - 1 = 0$, which can be approximated to $-4.51t^2 + 3.07t - 1 = 0$. The discriminant of this equation is $\Delta = 3.07^2 - 4\times 4.51\times 1 \approx -8.62$ and hence there are no solutions for time in that equation and the box never reaches the end of the ramp.