EL4

Mathematics Support Centre

Title: Superposition Theorem

Target: On completion of this worksheet you should be able to use the superposition theorem as an alternative method for circuit analysis.

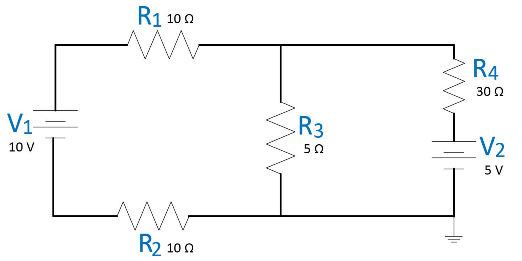
Introduction

The superposition principle states that for a linear circuit, containing more than one independent energy source, the voltage across or current through any circuit element is the algebraic sum of the voltages or currents due to each source applied separately with all other independent voltage and current sources removed.

This is useful for analysing complex circuits which consist of multiple current and voltage sources. Kirchoff's laws can still be used but the equations can be too complicated or difficult to solve.

Example

For the circuit below determine the current through resistor R_3 using the superposition method.

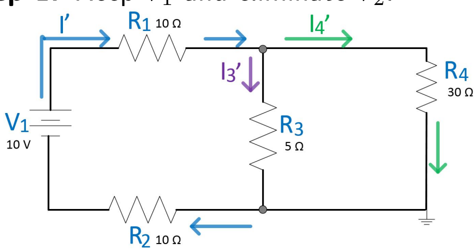


Solution

Considering the circuit consists of two voltage sources V_1 and V_2, there will be two main steps. We will keep V_1 and eliminate V_2 and vice-versa. The current through resistor R_3 will be determined in both cases. The final value of the current is calculated by adding up the two currents.

Note: Remember that the elimination of a voltage source is represented by a short-circuit.

Step 1: Keep V_1 and eliminate V_2.



Note: There are multiple ways of calculating the current through R_3.

Solution(Cont.)

Calculate $R_{eq'}$

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{5} + \frac{1}{30}$$
$$R_{34} = 4.286\ \Omega$$
$$R_{eq'} = R_1 + R_2 + R_{34} = 10 + 10 + 4.286$$
$$R_{eq'} = 24.286\ \Omega$$

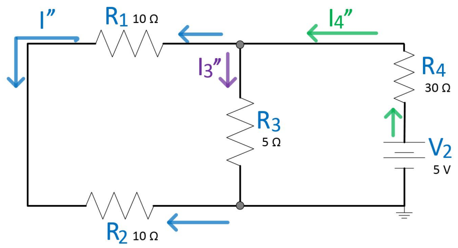
Calculate the supply current $I^{'}$:

$$I^{'} = \frac{V_1}{R_{eq'}} = \frac{10}{24.285} = 0.412 \ \text{ A}$$

Apply the Current Divider Rule for $I_3^{'}$:

$$I_3^{'} = I^{'} \times \frac{R_4}{R_3 + R_4}$$
$$I_3^{'} = 0.412 \times \frac{30}{5+30} = 0.353 \text{ A}$$

Step 2: Keep V_2 and eliminate V_1.



Calculate $R_{eq''}$

$$R_{12} = R_1 + R_2 = 10 + 10 = 20\ \Omega$$
$$\frac{1}{R_{123}} = \frac{1}{R_{12}} + \frac{1}{R_3} = \frac{1}{20} + \frac{1}{5}$$
$$R_{123} = 4\Omega$$
$$R_{eq''} = R_4 + R_{123} = 30 + 4 = 34\ \Omega$$

Calculate the supply current $I^{''}$

$$I^{''} = \frac{V_2}{R_{eq''}} = \frac{5}{34} = 0.147 \text{ A}$$

Apply the Current Divider Rule for $I_3^{''}$

$$I_3^{''} = I_4^{''} \times \frac{R_{12}}{R_3 + R_{12}}$$
$$I_3^{''} = 0.147 \times \frac{20}{5+20} = 0.118 \text{ A}$$

Solution(Cont.)

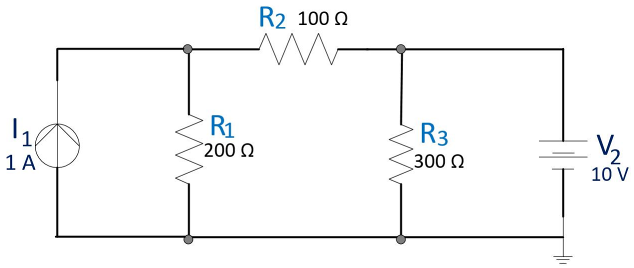
Step 3: Find the value of I_3

$$I_3 = I_3^{'} + I_3^{''}$$
$$I_3 = 0.353 + 0.118 = 0.471 \text{ A}$$

Note: The direction of the currents is important! If they are flowing in opposite directions then they must be subtracted.

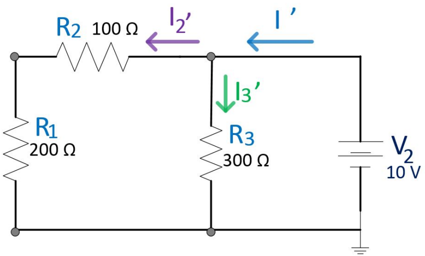
Exercise

For the following circuit calculate the voltage drop across R_1 using the superposition principle.



Solution

Step 1. Keep V_2 and eliminate I_1.



Method 1

Calculate $R_{eq'}$

$$\frac{1}{R_{eq'}} = \frac{1}{R_{12}} + \frac{1}{R_3}$$
$$R_{eq'} = 150\ \Omega$$

Calculate the supply current $I^{'}$:

$$I^{'} = \frac{V_2}{R_{eq'}} = \frac{10}{150} = 0.066 \text{ A}$$

Apply the Current Divider Rule for $I_2^{'}$:

$$I_2^{'} = I^{'} \times \frac{R_3}{R_{12} + R_3}$$
$$I_2^{'} = 0.066 \times \frac{300}{300 + 300} = 0.033 \text{ A}$$

Calculate the voltage drop:

$$V_{R_1^{'}} = I_2^{'} R_1 = 6.6 \text{ V}$$

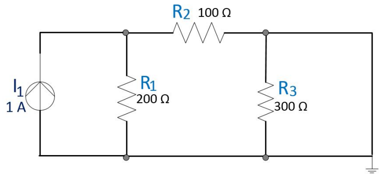
Method 2

Apply the voltage divider rule directly:

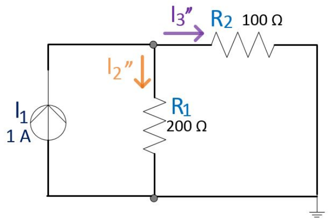
$$V_{R_1^{'}} = V_2 \times \frac{R_1}{R_1 + R_2} = 10 \times \frac{200}{200 + 100} = 6.6 \text{ V}$$

Solutions(Cont.)

Step 2: Keep I_1 and eliminate V_2.



Note: R_3 is short-circuited, therefore, it can be eliminated from the circuit.



Apply the Current Divider Rule for $I_2^{''}$

$$I_2^{''} = I^{''} \times \frac{R_2}{R_1 + R_2} = 0.334 \text{ A}$$

Calculate the voltage drop:

$$V_{R_1^{''}} = I_2^{''} R_1 = 66.6 \text{ V}$$

Step 3: Calculate the final value V_{R_1}

$$V_{R_1} = V_{R_3}^{'} + V_{R_3}^{''}$$
$$V_{R_1} = 79.8 \text{ V}$$