E4

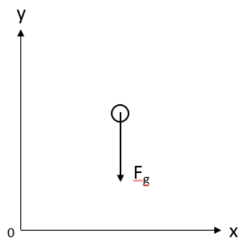
Mathematics Support Centre

Title: Forces and free-body diagrams

Target: On completion of this worksheet you should be able to sketch the free body-diagram for physical systems.

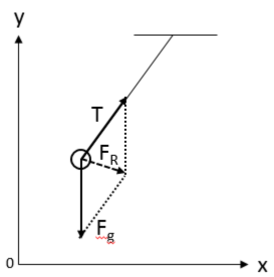
Forces

Force is an interaction that tends to change an objects motion. It is measured in Newtons. In SI units we have 1N=1kg·m/s^2. There exists a huge variety of forces. Force is a **vector**. In free-body diagrams, we mainly focus on the Hooke's force, $F_s = -kx(t)$, where k is the spring constant, the gravitational force $F_g = mg$, and the drag force $F_d = -cv(t)$, where c is the drag coefficient. Nevertheless, our considerations have applications for any type of forces. Graphically, we represent forces as arrows.



Free-body diagrams

In order to illustrate forces acting on a body we use a **free-body diagram**. The forces are represented by arrows attached to the body. The length of the arrows is proportional to the strength of the force and the tip of the arrows points towards the direction of the force. If multiple forces are acting on a body, then the total force will be the sum of all the forces. Keep in mind that force is a vector and therefore we need to sum the forces **component-wise**, by splitting the vectors into x, y and z components. Otherwise, we can use the **parallelogram** rule of vector addition, for example $\mathbf{T} + \mathbf{F_g} = \mathbf{F_R}$.



Newton's laws of motion in free-body diagrams

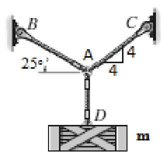
Now that we have established how to manipulate forces, we need a theory that will let us predict the results of our experiments. The three laws of motion developed by *Isaac Newton* come to rescue. *First law:* If there are no forces acting on a body, or the sum of the forces is equal to 0, the body will continue its motion with the same speed and the same direction, unless it was initially at rest. In that case it remains at rest. These two cases are called **equilibrium**.
Second law: If the sum of the forces acting on a body is not equal to 0, the body will accelerate, with the acceleration given by the formula $\mathbf{a(t)} = \frac{\sum \mathbf{F}}{m}$, where $\sum \mathbf{F}$ is the net sum of all the forces and m is the body's mass.
Third law: In the equilibrium state, if a body a exerts a force on another body b, then the body b exerts a force on body a with a force equal in magnitude but with the opposite direction.

Exercises

1. A ball of mass $m = 0.625$ kg is lying on the ground.
   a) What is the magnitude of the gravitational force acting on the ball?
   b) In addition, the Earth's surface is acting on the ball with a **normal force**. Using Newton's 3rd law find its magnitude and direction.
   c) Sketch the free-body diagram for this situation. Include the net force.

2. A child of mass 30 kg is sitting on a sleigh of mass 2 kg. His father attaches a rope to the sleigh and pulls him with the force $\mathbf{F}$ at the angle of $\theta = 30°$ with the surface. The sleigh is accelerating at 1 m/s^2 on a frictionless surface. ($g \approx 9.81$m/s^2).
   a) Find the horizontal component of $\mathbf{F}$.
   b) Sketch the free body diagram for the sleigh
   c) Find the magnitude of $\mathbf{F}$, the vertical component of $\mathbf{F}$, and the normal force due to Earth's surface.

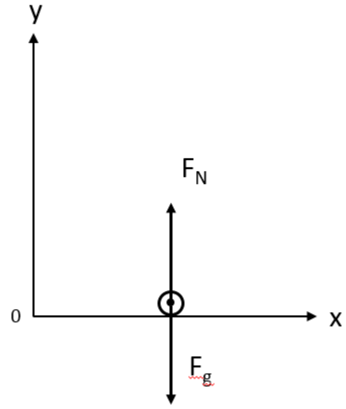
3. Box shown in the figure below has a mass of $m = 100$ kg and is supported by cables. Determine the magnitude of the force in each cable, take $g = 9.81$ m/s^2.



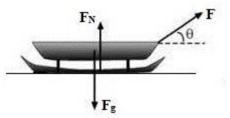
P.T.O.

Answers

1. a) $F_g = mg \approx 6.25$ N.
   b) Since the ball is at rest $|\mathbf{F_n}| = |\mathbf{F_g}| \approx 6.25$ N. It points in the opposite direction of the gravitational force, $\mathbf{F_n} = -\mathbf{F_g}$.
   c) The net force is equal to zero.



2. a) $F_x = ma = 32$ N.
   b) (Alternatively you can denote the sleigh by a point object.)

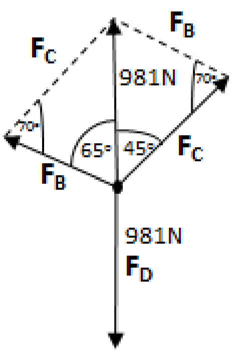


   c) $F_x = |\mathbf{F}|\cos(\theta)$. So, $|\mathbf{F}| = 36.95$ N. Then $F_y = |\mathbf{F}|\sin(\theta)$. So, $F_y \approx 18.48$ N. Finally $F_n = F_g - F_y \approx 295.44$ N.

3. $F_D = mg = 981N$.

$$\begin{cases} F_C\cos(45°) - F_B\cos(25°) & = 0 \\ F_C\sin(45°) + F_B\sin(25°) & = 981 \text{ N.} \end{cases}$$

   $F_B = 738$ N, $F_C = 946$ N.
   Alternatively, as $\mathbf{F_B} + \mathbf{F_C} = -\mathbf{F_D}$, we could apply the parallelogram method. We need to find the angles between the force components.



   Using the sine rule, we get $F_B = F_D\frac{\sin(45°)}{\sin(70°)} \approx 738$N, $F_C = F_D\frac{\sin(65°)}{\sin(70°)} \approx 946$N.