E7

Mathematics Support Centre

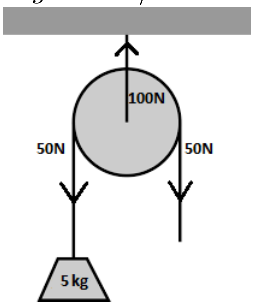
Title: Pulleys

Target: On completion of this worksheet you should be able to analyse pulleys systems.

Single fixed pulley

In the simple pulleys we assume that
1. *The lines are weightless and don't stretch.*
2. *There is no friction between ropes and the blocks.*

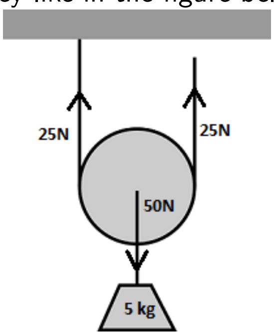
This allows us to conclude that in equilibrium, **the tension is the same on both sides of the pulley**. The diagram below shows all the forces acting on a simple pulley. Notice that the net force has to be zero in order to keep the pulley in equilibrium. For simplicity we take $g \approx 10\text{m/s}^2$.



The role of the pulley is only to **change the direction** of the force needed to raise the weight. The magnitude of the force needed to raise the weight remains **the same**.

Single moveable pulley

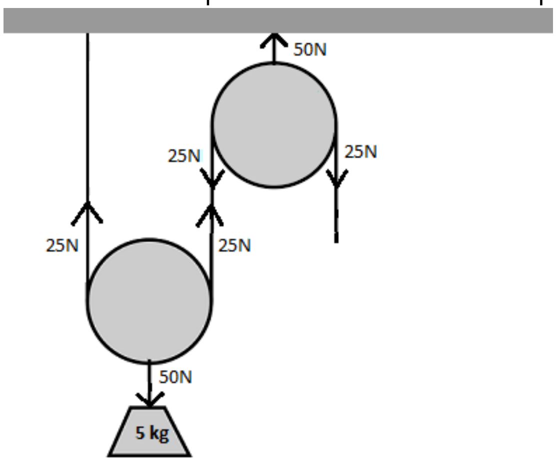
What happens when we 'invert' the mechanism and construct a pulley like in the figure below?



This pulley **does not change the direction of the force**, since we're still pulling the weight up. However, it **halves the magnitude of the force** required to pull the weight up. How is that possible? Even though the force is reduced, the overall 'effort' required to pull the weight up remains the same, since we'll be pulling twice the amount of rope as the pulley and the weight move up.

Combined pulleys

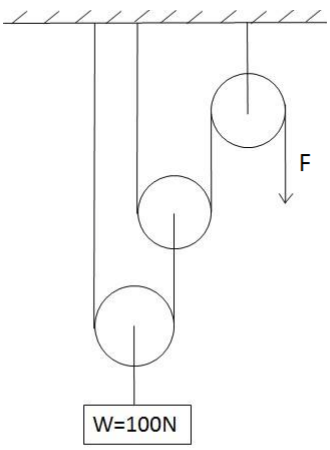
Basing on the same principles we can combine 2 or more pulleys together, even those of different type. We need to remember that the sum of the forces 'pointing up' should be equal to the sum of the forces 'pointing down' if the system is in equilibrium. Otherwise, the system would accelerate as a whole (refer to Newton's second law). To make sure that the pulley remains in a **rotational equilibrium** we need to make sure that the tension is equal on both sides of the pulley.



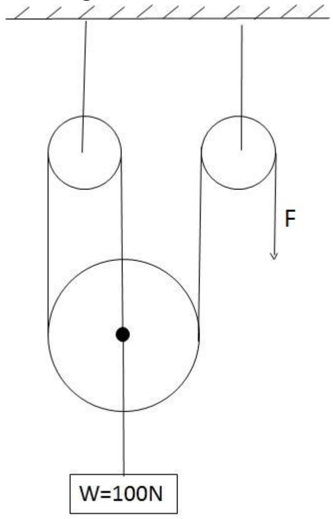
Exercises

1. Find the force required to keep the system in equilibrium for the following systems.

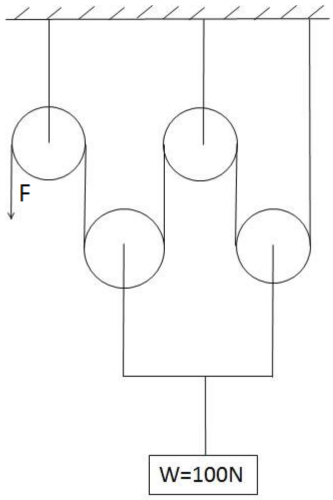
   a)



b) (The weight is attached to the big block)



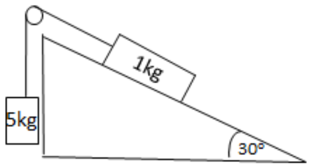
c)



2. In the system 1 a) find the acceleration of the load if the force pulling the ropes is 100N. ($g = 9.81$m/s^2).

3. By using the principles shown in 1 a), it is possible to reduce the force required to lift the load drastically. In particular, if n moveable pulleys are attached together, the force required to lift the load is reduced by a factor of 2^{-n}. What is the side effect of this system which stops us from constructing chains of n pulleys attached this way?

4. The coefficient of friction between the ramp and the block is $\mu_k = 2$. Find the magnitude of the acceleration of the system. Take $g \approx 9.81$m/s^2.



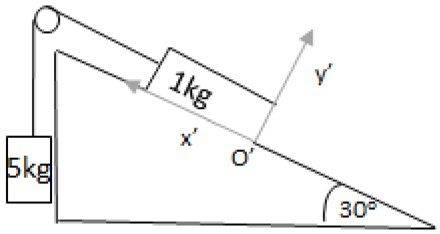
Answers

1. a) F=25N.
   b) F=33N.
   c) F=25N.

2. The mass of the object is $m = \frac{W}{g} \approx 10.19$kg. Since the force required to keep the system in equilibrium is 25N, we have the net force of $F = 100$N-25N$=$ 75N and therefore $a = \frac{F}{m}$=7.36m/s^2.

3. To compensate for the decrease of strength required to lift a body up, the amount of rope required to pull x increases so that the product of Fx remains constant. For n pulleys we have an increase of 2^n rope length!

4. The magnitude of the acceleration will be the same for the weight and the block on the ramp. We will calculate it in the coordinate system shown below



We take the mass of the weight to be m_1 and the mass of the block to be m_2. Since the magnitude of the tension is the same on both sides of the pulley and the motion is 1D we have:

$$m_1 g - m_2 g \sin(\alpha) - m_2 g \mu_k \cos(\alpha) = (m_1 + m_2) a$$

$$a = \frac{m_1 - m_2 \sin(\alpha) - m_2 \mu_k \cos(\alpha)}{m_1 + m_2} g$$

$$a \approx 4.5\text{m/s}^2$$